

M4X00391

TEST 1

SOLUTION

JAN 2021

Question one

a
i

$$y = 4x^4 + 5x^3 + 2x^2 - 3x + 9$$

$$\frac{dy}{dx} = 16x^3 + 15x^2 + 4x - 3$$

②

ii

$$y = 5(2x+4)^3$$

$$\begin{aligned}\frac{dy}{dx} &= 15(2x+4)^2 \cdot 2 \\ &= 30(2x+4)^2\end{aligned}$$

②

iii

$$y = \sqrt{x} + \ln x$$

$$\frac{dy}{dx} = \frac{1}{2}(x)^{-\frac{1}{2}} + \frac{1}{x}$$

②

iv

$$y = \sin 5x + 6 \cos 2x + 3 \tan 3x$$

$$\frac{dy}{dx} = 5 \cos 5x - 12 \sin 2x + 9 \sec^2 3x$$

③

v

$$y = x^3 \cos x$$

$$u = x^3$$

$$\frac{du}{dx} = 3x^2$$

$$v = \cos x$$

$$\frac{dv}{dx} = -\sin x$$

④

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{dy}{dx} = -x^3 \sin x + 3x^2 \cos x$$

Question one

Q

vi

$$y = \frac{2x^2 + 3x}{e^{5x}}$$

(5)

$$u = 2x^2 + 3x$$

$$v = e^{5x}$$

$$\frac{du}{dx} = 4x + 3$$

$$\frac{dv}{dx} = 5e^{5x}$$

$$y = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$$

$$\frac{dy}{dx} = \frac{e^{5x}(4x+3) - 5(2x^2+3x)e^{5x}}{(e^{5x})^2}$$

vii

$$y = e^{2x} \sin 2x$$

$$u = e^{2x}$$

$$v = \sin 2x$$

$$\frac{du}{dx} = 2e^{2x}$$

$$\frac{dv}{dx} = +2 \cos 2x$$

(4)

$$\frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$$

$$\frac{dy}{dx} = e^{2x} (+2 \cos 2x) + 2e^{2x} \sin 2x$$

$$\frac{dy}{dx} = 2e^{2x} \cos 2x + 2e^{2x} \sin 2x$$

Question One

b

$$z = 3x^6 + 5x^2y^4 + 2y^3$$

$$\frac{\partial z}{\partial x} = 18x^5 + 10y^4x \quad \frac{\partial z}{\partial y} = 20x^2y^3 + 6y^2$$

$$\frac{\partial^2 z}{\partial x^2} = 90x^4 + 10y^4 \quad \frac{\partial^2 z}{\partial y^2} = 60x^2y^2 + 12y$$

$$\frac{\partial^2 z}{\partial y \partial x} = 40xy^3 \quad \frac{\partial^2 z}{\partial x \partial y} = 40xy^3$$

(6)

c

$$y = 4x^3 + 5x + 6$$

$$\frac{dy}{dx} = 12x^2 + 5$$

$$\frac{d^2y}{dx^2} = 24x$$

$$2\left(\frac{d^2y}{dx^2}\right) + 3\left(\frac{dy}{dx}\right) - 4x - 8$$

$$2(24x) + 3(12x^2 + 5) - 4x - 8 =$$

$$48x + 36x^2 + 15 - 4x - 8 =$$

$$36x^2 + 44x + 7 = 36x^2 + 44x + 7$$

(4)

Question One

d

$$y = 3\cos 2x$$

$$y' = -6\sin 2x, \quad y'' = -12\cos 2x$$

$$y''' = 24\sin 2x, \quad y^{(4)} = 48\cos 2x$$

②

e

$$y = x^2 - 6x + 5$$

$$y = x - 3$$

$$x^2 - 6x + 5 = x - 3$$

$$x^2 - 6x + 5 - x + 3 = 0$$

$$x^2 - 7x + 8 = 0$$

$$x = \frac{7 \pm \sqrt{7^2 - 4 \times 1 \times 8}}{2}$$

$$x = \frac{7 \pm 4.12}{2}$$

$$x = 5.56$$

$$x = 1.438$$

$$y = 2.56$$

$$y = -1.56$$

⑥

Question two

$$\ln(3x-5) + \ln(3x+4) = 2\ln(3x-4)$$

$$\ln(3x-5)(3x+4) = \ln(3x-4)^2$$

(5)

$$(3x-5)(3x+4) = (3x-4)(3x-4)$$

$$9x^2 + 12x - 15x - 20 = 9x^2 - 12x - 12x + 16$$

$$9x^2 - 3x - 20 = 9x^2 - 24x + 16$$

$$21x = 36$$

$$x = \frac{36}{21} = \frac{12}{7}$$

$$3^{2+x} \times 2^{3-x} = 5^x$$

$$\log(3^{2+x} \times 2^{3-x}) = \log 5^x$$

$$\log 3^{2+x} + \log 2^{3-x} = \log 5^x$$

$$(2+x) \log 3 + (3-x) \log 2 = x \log 5$$

$$(2+x) 0.48 + (3-x) 0.3 = x (0.7)$$

$$0.96 + 0.48x + 0.9 - 0.3x = 0.7x$$

$$-0.52x = -1.86$$

$$\boxed{x = 3.58}$$

(4)

Question two

Q

III

$$\frac{4}{2x+5} = \frac{6}{x+2}$$

(3)

$$4(x+2) = 6(2x+5)$$

$$4x + 8 = 12x + 30$$

$$4x - 12x = 30 - 8$$

$$-8x = 22$$

$$x = \frac{-22}{8} = \frac{-11}{4}$$

$$x = -2.75$$

IV

$$\frac{x+2}{6} + \frac{7x+1}{9} = \frac{x}{18}$$

(4)

$$18 \frac{x+2}{6} + 18 \frac{7x+1}{9} = 18 \frac{x}{18}$$

$$3(x+2) + 2(7x+1) = x$$

$$3x + 6 + 14x + 2 = x$$

$$16x = -8$$

$$x = -\frac{8}{16}$$

$$x = -\frac{1}{2}$$

Question two

b
i

$$t^2 + 8t + 15$$
$$(t+5)(t+3)$$

②

ii

$$5q^2 + 17q + 6$$
$$(5q+2)(q+3)$$

②

iii

$$N^2 - 121$$
$$(N-11)(N+11)$$

②

iv

$$m^2 - 4mn + 3m - 12n$$
$$m(m-4n) + 3(m-4n)$$
$$(m-4n)(m+3)$$

③

Question two

i
 $6x^2 + 19x + 15 = 0$

$$(3x+5)(2x+3) = 0$$

$$3x+5=0, \quad 2x+3=0$$

$$x = -\frac{5}{3}, \quad x = -\frac{3}{2}$$

③

ii
 $x^2 = -2x + 24$

$$x^2 + 2x - 24 = 0$$

$$(x-4)(x+6) = 0$$

$$x = 4 \text{ or } x = -6$$

②

iii
 $2x^2 - 21 = 11x$

$$2x^2 - 11x - 21 = 0$$

$$(2x+3)(x-7) = 0$$

$$2x+3=0, \quad x-7=0$$

$$x = -\frac{3}{2}, \quad x = 7$$

③

d

Question two

$$6x^2 - 13x - 8 = 0$$

$$x = \frac{-(-13) \pm \sqrt{(-13)^2 - 4(6)(-8)}}{2(6)}$$

$$x = \frac{13 \pm \sqrt{169 + 192}}{12} = \frac{13 \pm \sqrt{361}}{12}$$

$$x = \frac{13 \pm 19}{12}$$

$$x = \frac{8}{3} \quad \text{or} \quad x = -\frac{1}{2}$$

(4)

e

$$T = 2\pi \sqrt{\frac{L}{mgh}}$$

$$T^2 = 4\pi^2 \frac{L}{mgh}$$

$$4\pi^2 L = T^2 mgh$$

$$m = \frac{4\pi^2 L}{ghT^2}$$

(3)

Question three

a

$$\frac{5x-3}{(x-5)(x+6)} = \frac{A}{(x-5)} + \frac{B}{(x+6)}$$

$$(5x-3) = A(x+6) + B(x-5)$$

$$\text{let } x = -6$$

$$5(-6) - 3 = B(-6-5)$$

$$-33 = -11B$$

$$B = \frac{33}{11}$$

$$B = 3$$

$$\text{let } x = 5$$

$$5x-3 = A(x+6) + B(x-5)$$

$$25-3 = A(5+6)$$

$$22 = 11A$$

$$A = \frac{22}{11}$$

$$A = 2$$

$$\frac{5x-3}{(x-5)(x+6)} = \frac{2}{(x-5)} + \frac{3}{(x+6)}$$

(10)

Question three

i
 $\sin x = 0.6$

$$x = \sin^{-1} 0.6$$

$$x = 36.9^\circ$$

$$x = 143.1^\circ$$

②

ii
 $7 \cos x = -5$

$$\cos x = \frac{-5}{7}$$

$$x = \cos^{-1} \frac{5}{7}$$

$$x = 44.4^\circ \text{ not a solution}$$

$$x = 224.4^\circ$$

$$x = 135.6^\circ$$

④

iii
 $\tan x = 0.85$

$$x = \tan^{-1} 0.85$$

$$x = 40.4^\circ$$

$$x = 220.4^\circ$$

②

Question 3

$$BC^2 = 12^2 + 15^2 - 2(12)(15)\cos(39^\circ)$$

$$BC^2 = 89.2 \text{ cm}^2$$

$$BC = 9.44 \text{ cm}$$

$$\frac{9.44}{\sin 39^\circ} = \frac{12}{\sin C}$$

$$\sin C = \frac{12 \sin 39^\circ}{9.44}$$

$$\sin C = 0.8$$

$$C = 53^\circ$$

$$B = 180^\circ - (53^\circ + 39^\circ)$$

$$B = 88^\circ$$

$$2 \log_b x + 3 \log_b y$$

$$= \log_b x^2 + \log_b y^3$$

$$= \log_b x^2 y^3$$

⑤

③

②

④

Question three

$$x^2 - 12x + 35$$

$$(x+7)(x+5)$$

$$(\tan x + 7)(\tan x + 5) = 0$$

$$\tan x = -7$$

$$\tan x = 7$$

$$x = \tan^{-1} 7$$

$$x = 81.9^\circ \text{ not answer}$$

$$x = 98.1^\circ$$

$$x = 278.1^\circ$$

$$\tan x = -5$$

$$\tan x = 5$$

$$x = \tan^{-1} 5$$

$$x = 78.7^\circ \text{ not answer}$$

$$x = 101.3^\circ$$

$$x = 281.3^\circ$$

(8)